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THE 'HOMING HABITS' OF THE PULMONATE MOLLUSK ONCHIDIUM[1]

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For some time it has been known that the limpets and their allies living on shore rocks exposed by the falling tide exhibit the habit of returning, on the approach of the period of low water, to particular spots where they have individually made 'homes,' slight depressions in the rock-surface, which usually fit with considerable exactness the form and irregularities of the limpet's shell. From this 'home' the limpet may creep away 50 to 90 cm., when covered by the sea, to feed—frequently following well-defined paths, to which it also adheres when creeping homeward again. Such experimental work as has been directed to the study of this peculiarity has gone to show that it may be regarded as depending upon tactile and perhaps other forms of irritability (Bohn, Piéron), the mollusk remembering (possibly) a sensory map of the topography of its home region. It cannot be said, however, that this matter is as yet thoroughly understood. 'Homing' behavior of a similar sort has also at various times been attributed, in an anecdotal way, to different terrestrial slugs and snails.

It seems previously to have escaped attention that homing movements of a more striking character are to be found in another gastropod, the intertidal naked pulmonate Onchidium. We have studied in this respect the behavior of Onchidium (Onchidella) floridanum Dall, and although further investigation of the subject has been planned, it is thought well to record here the nature of certain of our findings.[2]

The homing phenomenon to which we refer may be characterized as follows: Onchidium lives in groups, or communities, numbering up to a dozen or more individuals in each, which during high tide find shelter in cavities within the eroded shore rock; these cavities are either narrow crevices, or may be subspherical with a diameter as great as 6 cm.; but in any case they communicate with the exterior by means of a small and almost undetectable opening

(usually single), commonly obstructed by the growth of small mussels (Modiolus). When, during daylight hours, the tide has fallen to such an extent that an Onchidium 'nest' has been above water-level for about half an hour, which frequently happens about two hours before extreme low tide (but, for many nests, does not occur at all during neap tides), the Onchidia in that nest creep out, in succession, wander some distance over the algae-covered rocks, on which they feed, and then (at least an hour before their nest would again be covered by the rising of the tide) all return to that nest. The period of exposure lasts about two hours. Any one community may include individuals of varying lengths up to 2 cm.

The Onchidia return each to its own particular nest, even on much-eroded rocks and where the nests are many and close together, and that in spite of the fact that the individuals emanating from different nests frequently mingle during their wanderings. The snails comprising anyone community begin the return to their nest almost simultaneously, although they may be scattered over an area a meter in diameter and may be situated on opposite sides of the nest.

Clearly, at least two distinct problems are presented by this behavior: First, why do the Onchidia return to the nest, and above all, why do the various members of a group begin almost simultaneously to make this return? Secondly, what is the nature of the directive control of the return journey itself? For the present, we discuss only the latter problem.

Onchidia situated at some distance from their nest pursue a fairly direct path toward it when they begin to return; previously, their paths may have been quite irregular. One or more approximately linear depressions may be adhered to in the immediate vicinity of the nest itself, but this is not necessary. An individual taken from one side of its nest, and placed on the opposite side, at a distance of a meter or slightly more, frequently, exhibits little or no hesitation in turning and moving directly toward the nest.

The possibility of vision of the entrance to the nest, of heliotropic orientation, or of wind influence, severally or in combination, can be perfectly excluded from the rôle of directive agents in this matter. Neither can the wetness nor dryness of the substratum or of the animals' tissues be important, for the behavior of the snails is the same during a dry and torrid afternoon as in a drenching down-pour of rain.

In an intensively studied situation where a number of Onchidium nests were found close together (30 to 50 cm. apart), it was seen that an Onchidium of one community so placed as to creep across the tiny sunken gully followed by the members of a neighboring colony would sometimes after hesitation, take this path and follow it for some centimeters; but in only one out of ten such instances did it actually *enter* the foreign nest; always there was hesitation and a retracing of the path, combined in several cases with an encircling journey about the foreign entrance. Yet, neither on the natural surfaces which they frequent, nor on various artificial surfaces tested does an Onchidium evince

any tendency to follow its own slime-track or that of another individual; nor does it 'favor' a wet or a dry surface, a rough or a smooth one, in any detectable way (as, for example, an earthworm does).

It may be considered that some form of contact irritability, perhaps resident in the oral lapets, is partially responsible for this behavior. Experimental tests of this point, which are difficult to secure, are not yet complete. If the path ordinarily followed in the immediate vicinity of the nest be scraped bare, or the rock at the very entrance to the nest be chiseled away, the Onchidia endeavoring to return there collect at the edge of the cleaned area and then wander about in its neighborhood until they are covered by the tide (and washed off the rock).

There are additional features, of the first importance: An Onchidium found returning to its nest may be carried 50 cm. away and placed on rock above high water level, where these animals never go naturally, and in at least half the trials the snail succeeds in getting back to its nest. If a nest to which the inmates have returned be broken open and the animals placed on the rock at distances up to 50 cm. from the former entrance, they have no serious difficulty in making their way to the location of the old home.[3] Onchidia from neighboring colonies, or from laboratory stock, make no attempt to enter such a nest.

We are therefore forced to the provisional opinion that an Onchidium returns to its particular nest by virtue of some internal condition, simulating memory of the position of this nest in terms of its surroundings, but independently of the guidance which may be afforded by mechanically directive features of the environment.

To the extent that the homing movements of Onchidium may be proved to involve associative memory, this snail may be placed in a series comprising such types as Chiton, Fisurella, Onchidium, and Octopus, all four of which, in a sense, exhibit homing behavior, but of increasing degrees of precision and complexity in the order of the arrangement here given. The further study of Onchidium, both for itself and in relation to these other mollusks, should give rise to some valuable conclusions.

Summary.—Onchidium floridanum lives during high tide in 'nests,' cavities in the rock containing a number of individuals, from which the mollusks at low water emerge to feed upon exposed shore surfaces. The individuals emanating from any one nest return simultaneously to that nest before the tide rises again, and during this return give evidence of homing behavior, which seems to depend mainly upon internal conditions akin to the remembrance of specific localities.

[1] Contributions from the Bermuda Biological Station for Research, No. 95.

[2] A more complete account of these observations, with supplementary material, will be found in our paper, 'On the behavior of Onchidium,' shortly appearing elsewhere.

[3] When a nest has two openings, the removed inmates, or some of them, may turn directly to the *second* opening and enter there, even if they have in the first instance employed the other entrance.

GROWTH AND DURATION OF LIFE OF CHITON TUBERCULATUS[1]

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Communicated by E. L. Mark, September 30, 1918

The common chiton of the Bermuda Islands, C. tuberculatus L., passes all of its post-larval and adult life within the limits of the tidal zone. A thoroughly representative sample of the total chiton populatlon is therefore obtainable, though with difficulty in some places. In certain specific locations it has been possible to study practically all of the chitons there existing. Individuals are of various sizes, ranging during the spring months from 0.6 to 10 cm. in length. The frequency distribution of the lengths of the chitons at each locality is of an obviously multimodal character. It seemed likely, then, that one could work out some of the main features of the growth and duration of life of this mollusk. Facts of this sort, of which in the natural history of marine invertebrates there is a conspicuous deficiency, are not only important in a purely biological way, but have also a bearing upon the problem of estimating in outline the general fertility of the sea. This paper embodies a preliminary report upon one phase of an investigation having to do with the ethology of the chitons, especially from the standpoint of analyzing their behavior under natural conditions.[2] In a subsequent paper it will be shown that the growth phenomena in chiton probably afford a basis for the consideration of important questions which it has been impossible to approach successfully in such growth-studies as have previously been made on some other organisms in the warmer seas (e.g., Madreporarians). In this paper data are summarized establishing mean growth-curves for chitons inhabiting the region of Great Sound, Bermuda; although the chiton environments comprised within this region, a semi-enclosed sound, are ethologically far from being of a strictly homogeneous type, they nevertheless agree in certain significant physical respects, wherein they differ from other classes of situations (e.g., shores washed by the ocean surf) where chitons of this species also live.

Each of the eight plates which make up the 'shell' of chiton is compounded of a superficial 'tegmentum,' containing sensory receptors (photoreceptors[2]), and of a more deeply situated 'articulamentum.' The central area of the tegmentum of each plate is marked by a series of more or less distinct transverse grooves, or 'growth-lines.' The number of these growth-lines, best counted upon the six intermediate plates, is constant upon the different plates of one individual. On the assumption that these growth-lines, analogous to the concentric markings of the scales and otoliths of teleosts,[3] are formed at the rate of one per year, it is possible to estimate the growth-rate of chiton. The probable correctness of this assumption is assured, (1) by the inspection, throughout the year, of the growth of the young chitons appearing after the

close of the breeding season; (2) by the direct observation that a growth-line is formed during the winter period, and (3) by the coincidence of the findings based upon the counts of growth-lines with those derived from the modes in the frequency-distribution of sizes in the chiton population.

The shell-plates are eroded at a characteristic rate, depending upon the particular environment, the erosion tending to obliterate the growth-lines. This difficulty in making the counts can, however, be overcome, and the measurement of the rate of the erosion itself affords an additional check upon the estimations.

From the region of Great Sound 1090 chitons were examined; of these the ages of 881 (including 476 mature males and 352 females) were determined from the shell-markings. The growth-curve (*A*) in figure 1 was constructed

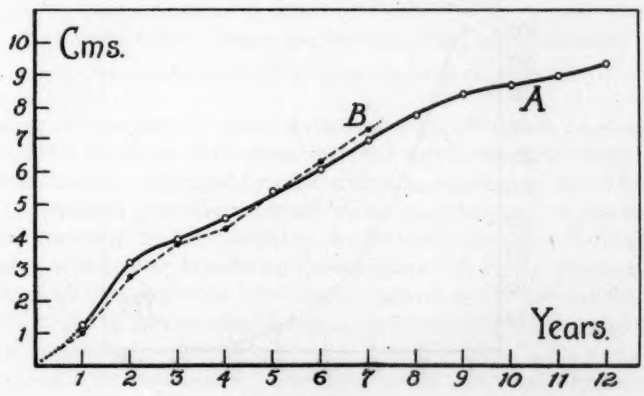

FIG. 1

FIG. 1. *A*, The modal lengths of Chiton tuberculatus at estimated ages one to twelve years; *B*, the growth of a chiton seven years old, as calculated from its shell markings. Great Sound, Bermuda.

from the modes of the lengths in the respective age classes. These modes were closely coincident with the associated means. The data are heterogeneous to the extent that chitons from different situations within Great Sound have been lumped together. The mean growth-curve for either sex is sensibly the same. The form of the mean growth-curve—exhibiting an initial increase in growth-rate followed by the appearance of several overlapping 'cycles' of growth (Robertson)—is similar to that which may be calculated from the markings upon the shell of a single individual (fig. 1, *B*).

The temporary depression in the rate of growth which commences with the third year is coincident with the onset of sexual maturity. This agrees with the similar correlation to be inferred from some brief statements by Heath[4] concerning the growth of several other genera of chitons.

It is of interest to determine the duration of life of chiton in different localities. In figure 2 are plotted the percentage frequency of the ages observed for chitons of both sexes. Chitons less than four years old are difficult to obtain in large quantities, because they live in crevices which are in many cases practically inaccessible. Therefore it is legitimate to consider only those individuals which are estimated to be more than five years old. Beyond this age, C. tuberculatus tends more and more strongly to come out into exposed situations. Therefore the relative proportions of the age-frequencies observed beyond the six to seven year-class are probably very nearly correct. Defining the 'mean death point' as the age at which there are already dead 50% of

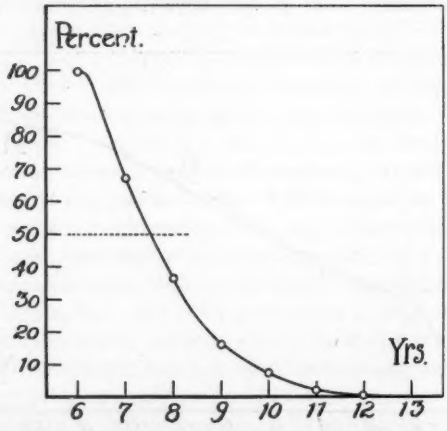

FIG. 2. THE PERCENTAGE SURVIVAL OF CHITONS AFTER THEIR SIXTH YEAR

Based upon records of 516 individuals six years old or over according to the indications of the shell markings. The 'mean death-point' is defined as that age at which 50% still survive; for this locality it falls in the eighth year.

those chitons found to attain the age at which dying-off begins, it is seen (fig. 2) that the natural termination of life comes on the average during the eighth year. This agrees, in general, with the estimated average age of shells found naturally dead.

The growth-curve established for the region of Great Sound, Bermuda, may be used as a norm with which to compare the growth of Chiton tuberculatus in other localities. These observations might with advantage be extended to more southerly portions of the West Indian faunal region, where Chiton tuberculatus is also found.

Summary.—A curve of the growth of Chiton tuberculatus has been obtained for the population in one locality at Bermuda, on the assumption that the age of a chiton may be estimated from the growth-lines upon its shell.

The mean duration of life is probably a little less than eight years. This curve affords a basis for comparing the growth of chitons in differing environments.

[1] Contributions from the Bermuda Biological Station for Research, No. 96.

[2] Cf. Crozier, W. J., *Amer. Nat., New York*, (in press), and Arey, L. B., and Crozier, W. J., (in press), for an account of observations forming the starting point of these investigations.

[3] Winge, Ö., *Medd. Komm. Havunders., Fisk., Kobenhavn*, **4**, No. 8, 1915.

[4] Heath, H., *Zool. Anz., Leipzig*, **29**, 1905, (390–393).

GROWTH OF CHITON TUBERCULATUS IN DIFFERENT ENVIRONMENTS[1]

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In the preceding paper a mean growth-curve for the chiton population of Great Sound, Bermuda, was derived from the examination of 1090 individuals, on the assumption, supported by collateral evidence, that the age of a chiton may be estimated from the markings upon its shell-plates. It was pointed out that curves of this kind might be used to study variations in the growth of Chiton tuberculatus in differing environments. A test of this suggestion is afforded by the comparison of the rates of growth and of the average duration of life of this species when living (1) in Great Sound, (2) on the south shore of Bermuda.

Even within the limits of Great Sound itself the chitons inhabiting the shores of the several islands may exhibit differing rates of growth, as indicated in figure 1. When two such islands as Long Island and Grace's Island are compared, it is found that the chitons from the former place grow somewhat less rapidly than do those from the latter, although the initial growth-rates are identical. Most of the Long Island chitons occur on the north shore of that island, which is a cliffed shore with many caves, sheltered from the sun but exposed to the beating of the sea during northwest winter gales. Grace's Island is more sheltered from storms.

On the other hand, chitons from an exposed beach on the south shore of Bermuda, such as that at Cross Bay, which supports a more numerous chiton population than do situations within the stiller waters of the Sound, grow at first more rapidly, but after the first year are smaller, as compared with those living in Great Sound.

The physical differences between two such environments as those presented by Long Island and by the beach at Cross Bay, include at least the following: the sea-water on the south shore is more alkaline, contains more oxygen, and is lower in temperature; the algal food for the chitons is not only specifically

less abundant on the south shore, but there are also relatively more of the chitons requiring it; at Long Island more shelter is afforded, in crevices, from the sun's heat at intertidal periods; at Cross Bay the older chitons, which live

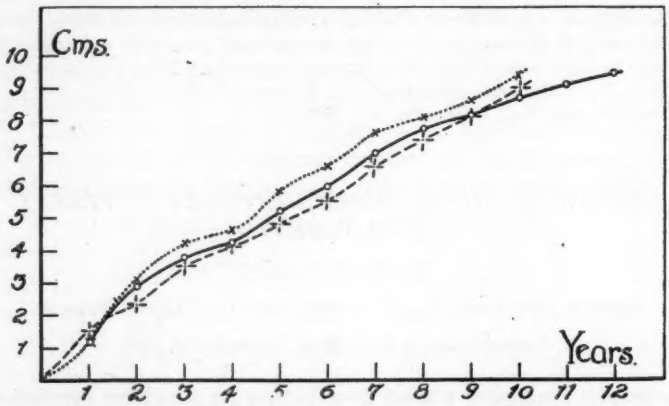

FIG. 1. THE RELATIVE GROWTH OF CHITON TUBERCULATUS IN DIFFERENT ENVIRONMENTS

—— o —— Long Island, Great Sound (422 individuals). ·····×····· Grace's Island, Great Sound (81 individuals). ——+—— Cross Bay, South Shore (591 individuals).

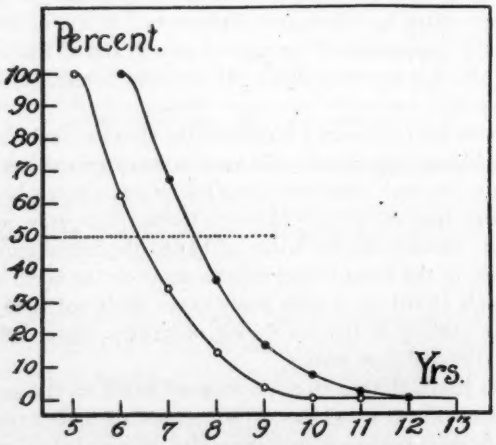

FIG. 2. THE PERCENTAGE SURVIVAL OF CHITONS ATTAINING THE AGE AT WHICH DYING-OFF BEGINS

● = Great Sound (515 individuals); o = Cross Bay, south shore (396 individuals).

further down the beach than do the younger ones,[2] are frequently covered to a depth of a foot or more by sand washed up by the tidal surf, and especially at times of southerly storms. The coöperation of these several differential

influences is reflected in the fact that the chitons at Cross Bay die more quickly than do those in Great Sound (fig. 2).

The death rate is not the same for the two sexes. According to the results embodied in figure 3, the female chitons, on the whole, tend to die off more rapidly than do the males. This results in a preponderance of males among the oldest members of the population.

Independently of this effect, however, there are apparent in the present data some curious variations in the proportion of the sexes within the successive age-groups. The fact that these variations have been found in two distinct localities favors the view that one is not dealing with chance fluctuation. With one exception[3] the Placophora are, so far as is known, dioecious, and in Chiton tuberculatus the sexes are readily distinguishable. To what

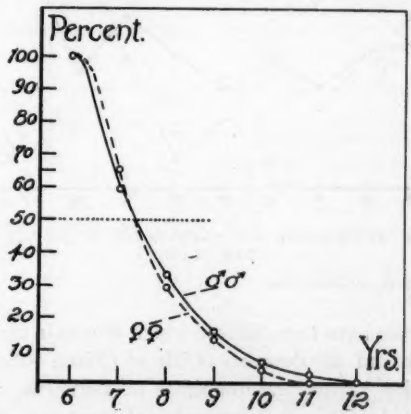

FIG. 3. THE RELATIVE DEATH-RATES OF MALES AND FEMALES (FROM RECORDS OF 912 INDIVIDUALS)

extent the fluctuations exhibited in the sex-ratio are due to environmental influences must be decided by further observation.

The sex of 1518 chitons was determined by dissection. The percentage of males ($R_{♂}$) in this series was 53.3; 1009 chitons from the region of Great Sound gave a ratio $R_{♂} = 54.9$; 509 from Cross Bay, a ratio $R_{♂} = 50.4$. With the data arranged to show the proportion of males at each age, in these localities, a progressive variation is evident (fig. 4). At first, there is a considerable preponderance of males, which, with fluctuations suggestively parallel in the two cases, is gradually decreased, until the differential death-rate becomes effective.

The chitons above seven years of age, which tend to occur in small groups scattered along the shore, exhibit a preponderance of males. Therefore it probably comes about, as a rule, that the eggs of a given female of this age or older stand a good chance of being fertilized, since solitary individuals of

maximum age are more apt to be males. Whatever of adaptational effect there may be detected in this arrangement is probably, however, not of great importance. The major share of the reproductive activity of the chiton population is carried on by individuals between five and seven years old; they are more numerous than the older ones, and the volume of the gonad is in them relatively greater than at other ages. The proportion of the sexes is here very nearly as 1:1. These animals are usually found close together in groups of a dozen or more, under stones or in crevices.

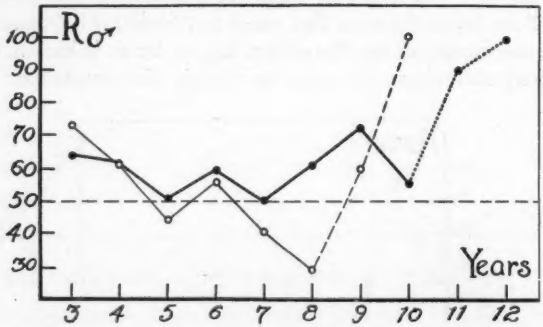

FIG. 4. VARIATIONS IN THE PROPORTION OF MALES IN DIFFERENT AGE-GROUPS OF CHITON TUBERCULATUS

● = Great Sound; ○ = Cross Bay.

Summary.—Curves have been derived which illustrate the extent to which the rate of growth and the duration of life of Chiton tuberculatus may be modified in differing natural environments at Bermuda; attention is also called to variations found in the proportion of the sexes, in different year-groups.

[1] Contributions from the Bermuda Biological Station for Research, No. 97.

[2] Cf. Crozier, W. J., and Arey, L. B., *Amer. Jour. Physiol., Baltimore*, **46**, 1918, (487–492). Arey and Crozier, (in press).

[3] Heath, H., *Zool. Anz., Leipzig*, **32**, 1907, (10–12).

THE INTERFEROMETRY OF VIBRATING SYSTEMS

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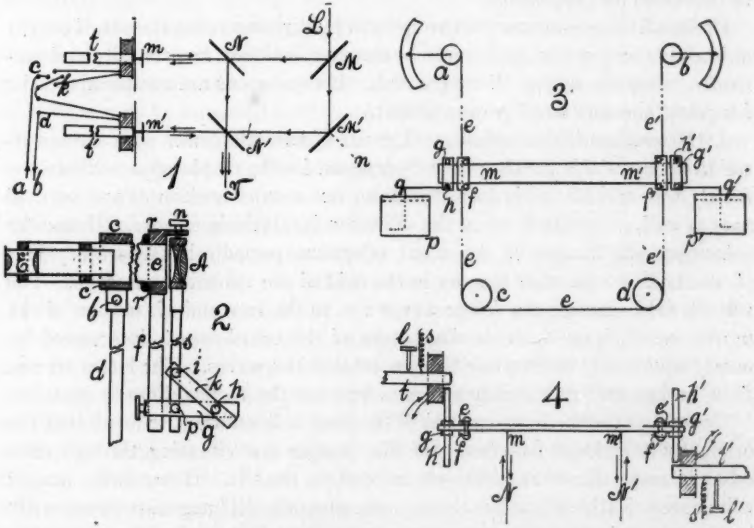
Communicated, September 6, 1918

1. Introductory.—The high luminosity of the achromatic interferences and the occurrence of but two sharp fringes, make it possible to utilize them even in cases when the auxiliary mirrors vibrate. Experiments of a similar kind were

tried with telephones and the spectrum ellipses before;[1] but these fringes do not easily admit of being drawn out into a ribbon and there is usually deficient light.

2. *Apparatus.*—I began the work with two similar telephones, as shown at t, t', in figure 1. Small mirrors were rigidly attached to the center of the diaphragms and each of the telephones secured on standards which admitted of adjustment around vertical and horizontal axes. The intermittent current was supplied at a, b from a small induction coil with a rheostat in circuit. Four clamp screws at c, d, were available for putting the telephone bobbins in series or in parallel. The current of one could be versed at k. White light, L, from a collimator, was reflected or transmitted by the half silvered mirrors, M, M',



N, N', of the interferometer and from m, m', on the telephones as indicated by the arrows. M' was on a micrometer with the screw on the direction, n. To facilitate the finding of the fringes one of the telephones t', should also be on a micrometer with the screw normal to m'. The fringes when found are observed by the vibration telescope at T.

The vibration telescope is shown in vertical section in figure 2, with the ocular at E and the objective originally as e, the tube being supported on the standard, d, and clamp cc, admitting of raising and lowering and of slight rotation around the horizontal axis, b. The objective, A, has been removed and is now supported by a flat steel spring, s, in front of its former position.

In order that the objective may vibrate parallel to the fringes and as these appear in all angles of altitude, the special vibratory system f, g, k, s, has been

devised. The rod *f*, reaching downward (about 10–15 cm. long) is attached to a ring of the objective, *rr*, capable of revolving with friction around the tube of the objective and of being fixed for any position in altitude of the rod. The horizontal clamp, *g*, adjustably attached to the rod, *f*, supports the spring, *s*. This passes through a vertical crevice in *g*; it is fixed by the set screw, *p*, and the two oblique adjusting strips, *k*, on either side of *g*. These pieces, *k*, are clamped by the screws at *h*, and serve as holders of the two coaxial set screws at *i*, on either side of *s*, so that the objective *A*, may be centered relative to the tube, *Ee*, for any oblique position of *f*, which must be normal to the fringes. The vibration may frequently be considerably changed by sliding *f* and *s* in *g* and reclamping the system. If the objective is to be fixed, a screw at *n* may be depressed for the purpose.

Under all circumstances the two spots of light representing the slit, if caught objectively on a screen at *T*, must be nearly coincident, horizontally and vertically, when the rays at *T* are parallel. If these spots are too far apart the fringes will be very small or even absent.

3. Observations with telephones.—The use of two telephones soon showed itself to be unsuitable for the present purposes, for the diaphragms oscillate not merely fore and aft (as in here desirable) but around horizontal and vertical axes as well, particularly when the vibration is relatively intense. Hence the coincident slit images of the silent telephone periodically separate. This shows itself in a peculiar manner in the field of the vibration telescope. The whitish field carrying the fringe waves due to the fore and aft motion of the mirror, *m*, *m'*, figure 1, on the diaphragm of the telephones, is intersected by nearly equidistant vivid white lines, normal to the waves. The latter are apt to be broken and appear only as traces between the vertical lines in question.

To account for these occurrences of the lines, it is sufficient to recall that the originally coincident identical fine slit images are vibrating through each other in some direction effectively normal to the slit. Hence these images will be seen clearly only at the elongations, when the slit images are temporarily stationary and at a maximum distance apart. Hence also the waves are absent at the lines, for here the slit images are so far apart as to eliminate the interferences.

As it made little difference how the telephones were electrically connected the diaphragms were removed and replaced by the two long strips of steel made from hack saw blades rigidly attached to the body of the telephone by a metallic arch. To approach this as near the magnet as possible forcing screws were provided at a little distance from the end of the strip. In this case the vibration of the strip and the mirror at its middle was fore and aft only and as a consequence the bright lines intersecting the field vanished completely. The arrangement of telephones, whether in series or parallel made a decided difference in the amplitude of the waves, which could be increased many times the breadth between successive fringes before the waves became turbulent and broke up.

4. Bifilar systems.—To utilize such a system as figure 1 to advantage it would be necessary to attune the elastic vibrators at m, m', to the same period, which should be as nearly as possible identical with the period of the source of intermittent or alternating current. As an earth inductor or a small magneto inductor (single magnet rotating in a flat coil) would have to be used in the latter case, it seemed best to convert the apparatus into a bifilar vibrator as shown in figures 3 and 4 in elevation and plan. Here mm' is a strip of thin mirror plate glass, about 32 cm. long, 1 cm. broad and 2 mm. thick, horizontal and in a position to receive the rays N, N' of the interferometer (compare figure 1). Motion of mm parallel to itself, fore and aft, will therefore produce no effect on the fringes; but any rotation around a vertical axis will be immediately apparent.

This strip of glass is supported by the bifilar system $e\,e\,e'\,e'$, made of a single thin wire. The ends of ee' are wound around the horizontal screws a, b which rotate with friction and are supplied with an index and scale, so that any amount of tension may be imparted to the wire. This passes below under the pulleys c, d, as nearly free from friction as possible, with the object of securing the same tensions throughout $e\,e'$. Flat clamps f, f' of fibre and screws, attach the strip mm' to the wire at any height, but necessarily near the middle of the vertical threads, where it receives the rays $N\,N'$.

The telephonic system consists of the horizontal soft iron screws h, h' similarly attached to mm' by the flat fibre clamps g, g', and the telephones t, t'. These were made of large flat files. The screws $l\,l'$ are used to approach the telephone magnets t, t', as near the soft iron armatures h, h' as possible, without overstepping the unstable position, in view of the tension of the tense wire e, e'. To give the vibrating system adequate damping, thin wires $q\,q'$ less that a millimeter thick, bent and dipping into lubricating oil in a small vat p, p' suffice. The fibres e, e' were about 45 cm. long and their distance apart about 29 cm. Their period and that of the vibrating telescope were made about the same, on the average about 0.2 seconds.

As a generator an earth inductor with a coil of wire 60 cm. in diameter was at first used but later replaced by a small magnetic inductor. It was turned by a small motor. To measure the average intensity of current a Siemens precision dynamometer was installed, indicating currents as low as one-tenth of an average milliampere. Periods from about 0.1 second to 0.3 second were available.

The three vibrating systems (mirror, telescope, alternator) thus all admit of an adjustment of their periods and these should be nearly the same if the elliptic system of Lissajous curves are to be obtained, which is the preferable case. A change of the tension of the wires $e\,e'$ in figures 3 and 4, or any adjustments at the telephones calls for a fresh search for fringes; but this is not difficult.

5. Further observations.—Without synchronism in the two vibrating systems (current and telephone) the motion of fringes obtained is practically inappre-

ciable when average currents within the order of milliamperes are treated. As soon, however, as approximate synchronism is established, the sensitiveness of the apparatus increases enormously. It is best for this purpose to vary the period of the motor of the alternate current generator by the slide rheostat If the fringes are horizontal and the objector therefore vibrating horizontally across the vertical slit image, the motion of he fringes is vertical. Hence the horizontal band of fringes in the absence of current, at once takes the form of a given Lissajous succession, with oppositions of rotations quite visible. The continuous change of these may be most conveniently accelerated or retarded by controlling the motor of the alternator with a slide rheostat. It is annoying if they leave the field while executing their gyrations, though they may always be restored by moving the micrometer. For mean tensions, the higher vibration curves 2 : 3, 3 : 4 may be obtained, both for the alternator moving at smaller and at larger periods than the vibrating mirror. To obtain them the motor running at maximum speed is gradually slowed down by means of the rheostat, when the forms appear in succession passing through the elliptic series at mean speeds.

The Lissajous curves continue to be very marked when additional resistances as high as 10,000 ohms are put into the circuit of the alternator. To obtain some idea of the smallest average current appreciable the Siemens dynamometer was put in circuit. Estimating the average current as $i = C\sqrt{\varphi}$ where C is the dynamometer constant and φ the deflection in centimeters, the current when 10,000 ohms is inserted and the magneto reduced in speed to $T = .25$ second was found to be of the average value of $i = 6 \times 10^{-6}$ amperes.

In a later and more refined adjustment the ellipses obtained with an insertion 10,000 ohms filled (as to their vertical or current axes) fully one-quarter of the field of the telescope. As little as one-tenth to one-hundredth of this would be easily appreciable with certainty, so that the minimum average current capable of detection may be estimated as well within 10^{-6} ampere. In this respect the device was somewhat disappointing. It must be remembered however that the above mirrors (mm') and appurtenances are unnecessarily heavy, and the bifilar too robust.

If one of the telephones is reversed, the fringes which showed marked vibration in the first position, frequently *ceases to show any vibration* until the ellipses in the former case are very large (small resistance in circuit). The results in such a case are uniformly consistent. The reason of this is apparent: For when the telephones are joined in series the lever mm', figure 3, is periodically rotated and released around a vertical axis and the displacement of fringes is proportional to the small angular amplitude of rotation. If, however, the telephones are connected differentially, the mirror mm', if properly adjusted, merely moves parallel to itself, fore and aft and the fringes remain stationary. More usually, however, there is a difference in the size of ellipses in the two cases.

It is for the investigation of this question that the adjustment pushing screws ll' and springs ss' (fig. 4) pulling toward the rear, the telephones being on a

vertical axis, were provided. If one of these, t for instance, is placed in a definite effective position, while t' is relatively far from its armature h', the screw l' may be gradually pushed forward diminishing the distance to the minimum. The effect of this is further to rotate mm', and if the turning of l' is cautiously done the fringes may be passed from top to bottom of the telescopic field by l' and restored to position by the micrometer. In this way it is possible to find the tension of ee' in which for one position of the commutator there is excessive motion of fringes whereas for the other there is practically no motion.

Adjusting the bifilar as to tension, there is one position or distance between h' and t' pretty sharply determinable, for which the fringe bands change to stationary ellipses in the *absence* of all current. This peculiar result is at first puzzling, but since it is quite synchronous with the period of the telescope (stationary ellipse), it is obvious that the motion of the objective is the cause of the phenomenon and that the fibers are now in unison with its period. For distances h', t', greater or smaller the ellipses soon return to bands. The effect of the alternating current on the stationary ellipse is very beautiful. It now oscillates very much like a smoke ring for one commutator position, whereas it passes in an accentuated way through all phases for the other.

Utilizing the preceding adjustment giving ellipses and bands respectively in the two positions of the commutator, many experiments were made to detect a change of phase when a large inductance is placed on both sides of one of the telephones. But in none of the experiments thus far, was any difference discernable to be attributed to the presence of the inductance.

As a tuned system responding to definite periods only, the vibration interferometer is quite sensitive, provided the average currents are of the order of several microamperes. Between the types of compound vibration curves corresponding to frequency ratios of 4/3, 3/2, 1/1, 2/3, 3/4 there is usually an unbroken band of fringes. If the ratio of periods remains fixed, the vibration curve of course remains fixed, which is the usual sharp acoustic criterion. When the ellipses (out of tune) change continuously between lines of different inclination, the passage in one direction is often gradual whereas in the reverse or return direction it is almost sudden. Linear forms flop into linear forms as it were. No doubt this is related to the vibration of a bifilar system like the above, where the two ends are liable to vibrate alternately. When the average currents approach the order of 10^{-4} am., the bands became waves for all periods not excessively high or low.

[1] *Washington, Carnegie Inst., Pub.*, No. 149, pt. 3, § 123–125, 1914; cf., *Amer. J. Sci., New Haven*, 3, 1897, (107).

ON THE ESSENCE OF PHYSICAL RELATIVITY

By Sir Joseph Larmor

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Communicated, September 19, 1918

In an important investigation recently reported,[1] Mr. Leigh Page has determined an expression for the translatory force required to sustain an assigned varying velocity in an electrostatic system of a certain type, that namely which it has been usual to investigate as a model of an electron in analytical discussions. The expression contains terms involving the acceleration of the system and its time-gradients, but no term involving its velocity except in combination with these other quantities. The application of his formula to which Mr. Page restricts himself is to challenge a result that an isolated radiating system is subject to retarding force from the reaction of its own radiation of amount equal to its velocity multiplied by rate of radiation divided by c^2. The necessity for such refutation is based on the idea that on principles of relativity the velocity of an isolated body could have no meaning. But the formula obtained seems to leave this question as it was: for equally the acceleration of the isolated body could have no meaning; and moreover, though the thrust of the radiation is, in this case compensated, the velocity appears to be actually involved in the formula in the same manner as the impugned result would require for this particular problem of a convected electrostatic system, for which the radiation is extremely transient and very slight in the absence of extraneous force.

It has always seemed to me that this subject, which may be described as that of interaction of the æther with uniform motion, though of slight account phenomenally, is theoretically of high significance, in that it is destined perhaps to throw light on the nature of the forward momentum that is convected by radiation, and thence on the intimate dynamical nature of radiation itself and the physical function of the æther. I therefore propose briefly to indicate what I hold, provisionally of course, to be their present trend of knowledge. An adequate survey of this widely ramifying domain, in its various aspects, is difficult to compress, even when concentration and avoidance of detail is the main aim, and is therefore reserved for a communication to the *Physical Review*.

In the first place the plan now customary of regarding an electron as merely an aggregation of differential elements of electricity seems to be far too narrow a foundation for its internal properties, except by way of partial and provisional illustration. If electricity is to be explained as constituted of electrons, it can hardly be a step towards finality to explain electrons as constituted of electricity. On such a view the kinetic reaction of the inertia of an electron presents itself in theory under the artificial guise of a self-retarding force exerted

by the electron on itself, as given by Mr. Page's formula: moreover on this order of ideas there is no reason assignable why electricity should be aggregated into electrons at all, unless other forces and constraints extraneous to the electric theory are to be introduced to hold them together.

On the other hand an æther theory demands immediately the existence of electrons as the singularities in its own constitution—like vortex-rings in fluid —which determine its activity: and the electrons had already formed an essential part of electrodynamic theory before they were detected in the free state, and the actual magnitude of their inertia thereby ascertained in comparison with the masses of the atoms of the dynamical elements. As being thus self-subsisting mobile structures, whose interactions with each other at distances great compared with the dimensions of their nuclei are fully known, but whose internal constitution and adjacent influence could only be illustrated and guessed at, their dynamical specification will be most suitably, at any rate most hopefully, expressed in terms of their electric charge, their effective inertia, and their electrodynamic fields of known forms except inside and close up to the nuclei; all else being held in suspense as matter only for provisional speculation. As in the case of the simplest exemplar of a theory of a medium, the hydrodynamic theory for an ideal perfect fluid containing vortex rings or moving solids, this inertia may be aeolotropic: it may even when waves can travel be a function of the velocity instead of being constant. But it is to be the same whether the system which is its seat be moving with acceleration or not. The momentum of the system, of type mv, is not to involve its acceleration: nor is the energy of translatory motion of the body, of type $\frac{1}{2}m^1v^2$ where m^1 may be different from m. Otherwise all existing general dynamical theory would be dissolved. The production of acceleration is ascribed to applied force, which measured by its result as the value of $d(mv)/dt$. The formula for the force required to sustain an assigned varying motion of Mr. Page's special model of an electron does not in this way introduce only v and its acceleration f: the terms in it which involve the gradients of f have no place as yet in this scheme. Some origin extraneous to the electron must be assigned to them. On an æther-theory that cause is the radiation shot out from the electron while it is in varying motion, which convects somehow mechanical momentum away from it into the free state; the backward kick of this momentum acting on the source from which it is ejected is describable as the back-pressure of the radiation, which would of course exert a compensating forward kick on any obstacle, however distant, which absorbs it. However we may clothe our thought in language of relativity, it would appear that this issuing radiation does effectively possess an absolute velocity c, and therefore an absolute velocity of its source also is theoretically determinable from observations made in relation to it: and this seems to explain how it is that Mr. Page's formula for the force on a system entirely isolated can involve the acceleration of this system and its velocity also.

But a complete isolated system travelling with velocity v, when we include in the system all the radiation that has issued to however great a distance from its nucleus, cannot create new translational momentum. Therefore the backward pressure proportional to v of the radiational momentum projected from it namely $-v/c^2.dE/dt$ on this theory, must be compensated by disappearance of forward momentum attached to the source itself. The momentum of the source is mv: its rate of change is $mdv/dt+vdm/dt$; the first term is the usual kinetic reaction of the inertia m it has at the instant; the other term proportional to v is the compensating one here adumbrated. It follows that we must admit change of mass specified by the formulae $\delta m=\delta E/c^2$; so that the mass of a radiating body diminishes proportionally to the energy it radiates, except in so far as there is compensation provided by extraneous radiation which it absorbs.

For example, a particle of cosmic dust describing an orbit round the Sun is retarded by its own radiation, but its mass is kept up by the radiation it gains from the Sun: thus there is no compensation to the resisting force in this case, and the particle will gradually be sucked into the Sun, as Poynting in an important memoir showed.

The physical theory of electrodynamics, including optics, would thus, on the so-called classical lines—which can be held to be not yet obsolete, though special problems of expansion or development to adapt them to new experimental discovery are pressing —involve electrons specified in terms of charge and field and inertia, which are the link with matter, and it would involve also free radiation existing in the æther, specified as regards mechanical force by its convected secondary longitudinal momentum, but physically in evidence mainly by the type and energy of its transverse undulations. These are the materials out of which the science is to evolve itself: and the problem is whether that evolution can be gradually effected in a natural and consistent manner, and one which runs parallel with the course of evolution of experimental knowledge.

The true essence of the relativity of external knowledge is that we can investigate a system only in relation to some other system, and the most convenient perhaps the only feasible other system has been hitherto the ideal Newtonian frame of reference of space and time; for that is the canonical system, so to speak, with regard to which dynamical principles take on an ideal simple form, and it is a system which is being determined with continually expanding precision by the progress of astronomical science.

The special question now in evidence is—Is it now expedient to exchange this frame of reference, corresponding to c infinite, for another far more complex but very slightly different continuum having a finite space-time modulus c ? The more fundamental question is—Are we to assign to either frame dynamical properties, typified by propagation of physical effect in space in terms of undulations sustained by stress and inertia, or are we to assign to it properties solely geometrical and regard all physical effect as merely projected in duration across space? The forms of special unrestricted relativity which

have been recently current ultimately demand and perhaps prefer the latter course; and such modes of expression can apparently be elaborated so as to include most of optics, though perhaps in an artificial and unfruitful manner, if we replace the Newtonian scalar corpuscles of light by projected vector elements of fields of potential (not however conserved in value) from which forces would be analytically derivable. Mr. Page's work has improved very remarkably such a scheme of projected influence by showing that, provided c-relativity is postulated for space and time, it is elements of longitudinal electric force that may be regarded as projected from the sources, and are moreover conserved after the manner of *quanta* as they travel onward.

There is no absolute criterion to decide between the two ideals. The first order of ideas has proved itself as the foundation on which the interlaced fabric of electric and optical science has been actually constructed: the other seems to offer as yet only somewhat ingenuous and disjointed though significant expression for certain striking features of recent discovery which the former has not yet succeeded in assimilating, and seems to require us to obliterate the course of evolution of the science or perhaps to retain it as a mere historical survival.

All these modes of restatement of departments of physical science in more expressly relative terms may be comprehended as partial analytic developments of the far wider principle of the purely relational character of our external knowledge, which was advanced and systematically fortified with great abstract force in the general metaphysical domain by Bishop Berkeley; a principle by means of which he passed on to examine the criterion of real objective existence, and one which was well understood in its present aspect by his friends in Yale College nearly two centuries ago.

Thus in these matters we are hardly concerned with refuting any theory, for all are relative: it is fruitless to traverse any proposition, unless we take into account the definitions and context in which it subsists. The question is, as to which scheme of formulation gives as a whole the closest and most expressive representation of the complex of natural knowledge, and affords the most promising clue to its future elaboration and extension. But a choice does not by any means preclude development along other promising but provisional lines for which an interpretation has yet to be found. A main merit of Mr. Page's powerful papers, especially that of 1914, the feature which may be said to constitute them into a theory, is to my mind that they translate the usual so-called classical electrodynamic theory into the order of ideas, formulation under relativity of the c type being an essential feature, that describes physical action in terms of projected entities (which might perhaps even be *quanta*) of various kinds, effecting this translation in more intuitive and fundamental terms than had previously been attained to; and also that by their conciseness and geometrical directness they facilitate that comparison and contrast with the alternative order of ideas which is the essential matter.

[1] These PROCEEDINGS, 4, 1918, (47–46); *Physic. Rev., Ithaca*, (Ser. 2), 11, 1918, (376–400).

GRAVITATIONAL ATTRACTION IN CONNECTION WITH THE RECTANGULAR INTERFEROMETER

By Carl Barus

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1. Introduction.—The ease with which the rectangular interferometer admits of the measurement of small angles, induced me to adapt an apparatus with reference to it, for the measurement of the Newtonian constant. In addition to the usual system suspended in air, I also tested a floating system. Accurate work is scarcely to be expected in this laboratory where temperature variations and the agitation of the room conflict with the condition of its obtainment. But the trial is nevertheless interesting and the final work may be done elsewhere.

2. Apparatus.—There is room here only for the well known quartz fiber method of Boys, using a light needle, weighing when loaded but 1.9 grams. This was made of a rigid shaft of straw *mm*, figure 1, 26 cm. long, the ends being split slightly into four symmetrical segments each, which receive the two shots *m* and *m'* additionally secured with a little wax. The two light mirrors *b* and *c* were differently mounted: *b* on a fine pin *d*, snugly fitting corresponding perforations in the straw, was thus capable of rotation around the vertical axis (*d*) and moving up and down slightly. The mirror *c*, however, was mounted on a thin elastic strip of aluminum, clasping the shaft as shown in section at *e'*, *c'*. This is thus capable not only of rotating on a horizontal axis, but of being placed at different distances (moving right and left) from *d* to accommodate the rays of the interferometer, to which *b* and *c* are to be normal.

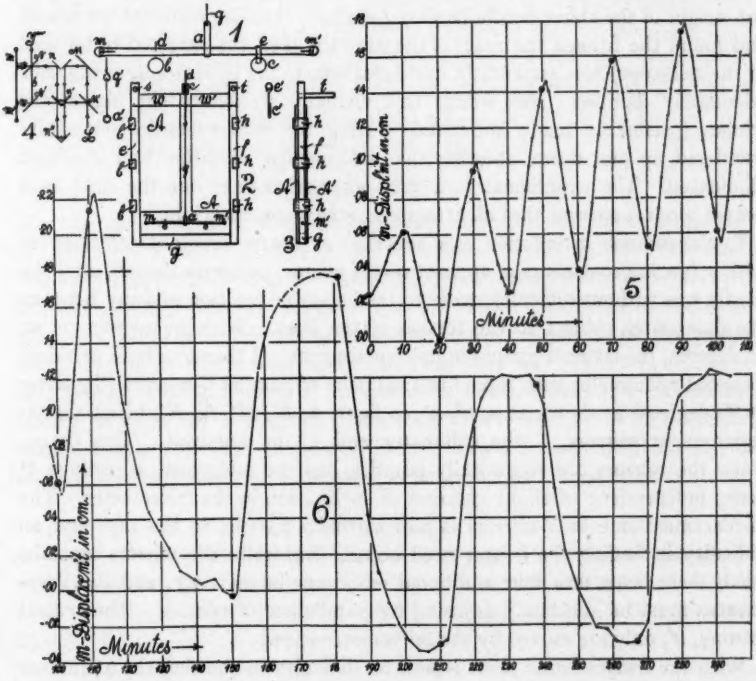
The needle is swung from a quartz fiber *q*, and a strip or hanger of elastic aluminum, *a* (see *a'*, *q'*), which clasps the shaft. Hence the latter may also be moved endwise to be balanced and rotated around a horizontal axis, till *b* and *c* meet the rays normally.

These operations are completed by trial before the needle is definitely hung, preferably in a broad beam of sunlight. No great accuracy is required.

The shallow case is made of two plates *AA*, figures 2, 3 (side view), of plate glass about 32 cm. broad and 45 cm. high, spaced by two strips *e*, *f*, 3 cm. wide of wood 1.2 cm. thick, reaching from top to bottom. Six steel clips, *b*, *b*, *h*, *h*, (such as are used for binding pamphlets), held these strips in place. At the projecting tops of the wooden strips, two nipples *s* and *t* of ¼ inch gas pipe fixed normally to the strips, served for the hanging of the case and needle from a firm wall bracket. The bottom of the case is subsequently to be closed from below by a strip of felting *gg*. To diminish the space within the case, two thin cloth covered wooden boards *w*, *w*, are inserted from the top.

The quartz fiber q to carry the needle mm, hangs from a long one-eighth brass rod d, which may be raised or lowered in view of the sleeve c held by a separate adjustable arm, without. The rod d must fit the perforation in the cork nicely, so that the former may be smoothly raised or lowered and held in any position in virtue of friction.

To swing the needle this is first placed on cork Y's, below the opening of the case AA, the felting gg having been removed. The quartz fiber q is then lowered on the long stem d, until the lower hook on the fiber is in position to grasp the clasp on the needle mm still below the case. The needle is then



cautiously lifted by raising d. After this the felt strip gg is inserted to close the case, and the necessary adjustment made at c and d to swing the needle freely in the restricted space provided for it.

In the rectangular interferometer used (fig. 4) the breadth of the ray parallelogram was $b = 10$ cm. and the angle of incidence $i = 45$ degrees, so that if the micrometer displacement ΔN corresponds to the small twist θ of the needle, $\theta = 0.071\ \Delta N$.

3. *Observations.*—The quartz fiber used (diameter $2r$) was $L = 17$ cm. long. Supposing the distance apart of attracting weights M and attracted weights

m, to be $d = 6$ cm., and $M = 10^3$ grams, if the rigidity of quartz is taken as 5×10^{11}, $\gamma = 6 \times 10^5 r^4 \Delta N$. Thus if

$r = 10^{-2}$ cm., $\Delta N = 1.1 \times 10^{-6}$ cm. per kg. of M, Period $T = 4.6$ sec.

$r = 10^{-3}$ cm., $\Delta N = 1.1 \times 10^{-3}$ cm. per kg. of M, $T = 465$ sec.

The first case would then correspond to but 1/40 of a fringe; in the second case there should be 250 fringes per attracting kilogram. The working conditions below proved to be much better than this, rigidity being estimated too high. If the tenacity of quartz be taken as 1.5×10^8 kg. per square centimeter the latter filament ($r = 10^{-3}$ cm.) should still hold 4.5 grams, or much more than the weight of the above needle (within 2 grams). Having mounted the needle and found the fringes, the image of the wide slit (i.e., the reflected beam seen in the telescope) was apparently quite stationary, the light needle being thus adequately damped. But within this virtually stationary slit image, the fringes (preferably made horizontal) during the winter months continually wandered up and down, showing that micrometric vibration had not been eliminated. The experiment is a very impressive one; but the drift in a heated room is so large that all attempt at measurement is idle.

In the summer installation in a subcellar at nearly constant temperature with a few improvements of apparatus (the mirrors being readjusted, etc.) the needle was without difficulty made to take a stable position midway between the glass plates, subject to the torsion of the fiber. With the micrometer at 45 degrees, the latter is limited in its displacement. I therefore later attached a special micrometer with three identical pairs of parallel V-mirrors (the latter at 90 degrees) as shown at n, n', n'', in figure 4, M, M', N, N', being the interferometer mirrors, L the collimator and T the telescope. This design when the mirrors are reciprocally parallel, has the additional advantage of being independent of slight changes of inclination in the micrometer. The displacement due to V-mirrors is now virtually parallel to the rays and no difficulty in finding the fringes need occur. Naturally the mirrors must be good, there being now four additional reflections in each ray; and the V-apparatus must be accurately adjusted for parallelism of mirrors. The central mirrors, n', only are moved by the micrometer screw.

With the fringes found there is now no difficulty in showing the attraction of gravitation. In fact in iron brick moved on a small truck near the shot at one end of the needle, gripped the balls m, m' very much like a magnet acting on the pole of a magnetic needle. Throughout the whole of the experiment the fringes were under the perfect control of the micrometer.

A more systematic experiment was made by testing the attraction of a lead ball 5.43 cm. in diameter and weighing about $M = 950$ grams, for the shot m' (at the end of the needle) weighing $m' = 0.61$ grams. The kilo was moved on a circular track with stops to a distance of $R = 4.24$ cm. (between centers of balls) from the ball of the needle, alternately. The period of the air-damped needle can not have been less than 18 min. The case is then that of a forced

vibration under constant force and a large logarithmic decrement. An example of the observations is given in the graphs figures 5 and 6, the reading being made every minute beginning with the equilibrium position (M in the neutral position). The ordinates of the graphs show the displacement of the attracted shot, m', in cm. The periods of alteration were 10 minutes during the first 110 minutes; thereafter the period of the needle was exceeded. Circles show the time of turning M, from one side of m to the other. The approach to a limiting arc is regular during the first 100 minutes, after which some extraneous disturbance enters; but both for the longer and the shorter intervals of alternation, the tendency is toward the same limiting arc. At the end of the damped period there is a sort of fluttering. The excursions of the needle are so large that it was not thought necessary to read the fringes throughout. In fact, if a micrometer scale is put across the wide slit, there are two methods simultaneously available for finding Δx, the displacement of m due to M. For if f is the focal distance of the collimator and Δy the reading in the telescope of the collimator micrometer scale specified,

$$\Delta x = \cos i \,.\, l\Delta N/b = \Delta y \,.\, l/2f$$

But the reading in terms of ΔN is enormously more accurate than the reading of Δy, the shift of the slit image in the telescope. If the motion of the needle were less nearly dead-beat, the attracting force could be computed from the limiting distance between elongations, if the torsion coefficient of the quartz fiber and the logarithmic decrement were known. From static experiments made during hour intervals this double amplitude was found to be 0.116 cm. or a departure of the shot m at the end of the needle from its position of equilibrium of 0.058 cm. in response to the attraction of M. If l is the semilength of the needle (between centers of shots), the micrometer displacement ΔN and the displacements Δx of the mass m are given by the equation

$$\Delta x = l\Delta N \cos i/b = 0.89 \cdot \Delta N$$

where b is the breadth of the ray parallelogram and $i = 45$ degrees the angle of incidence of the interferometer. Thus the micrometer displacement is of the same order as the displacement of m, and if the latter is 0.116 cm., we should have

$$\Delta N = 1.3 \text{ cm.}$$

As the micrometer reads to 10^{-4} cm. 1/1300 part of the attraction between M and $m = 0.61$ gram could therefore be detected; i.e., the attraction of 0.73 gram or per interference fringe well within one-third of this, for the given quartz fiber (which was not specially selected) and given distance $R = 4$ cm. This is equivalent to the attraction of 0.2 gram at a distance of 1 cm., per fringe.

Apart from the measurement of the torsion coefficient of the fiber, there is however a real difficulty involved, and that is the occurrence of marked drift in the needle. It is only incidentally that the fringes are found at rest. The chief contributory cause of this is no doubt the occurrence of motion of air

around the needle provoked by small differences of temperature, resulting for instance from illumination. If the possible accuracy of deflection measurement is to be of any value, therefore, the apparatus must be kept in the dark except during observation. Fortunately the achromatic fringes require little light. Even then a closed case which can be exhausted of air is essential, for such radiometer forces as may enter would in any event be differential, seeing that the mirrors are symmetric and the illumination is not subject to alternations like the forces on m. It is probable that in case of the above regular method a thicker quartz fiber and a greater distance R would conduce to the best results since ΔN is the least difficult quantity to determine.

The question may now be asked whether it is not possible in the present case to determine gravitational attraction in terms of the acceleration of balls resulting. With an ocular micrometer this would not be difficult as the fringes move slowly enough that the position can always be sharply specified.

But with a needle of long period in vacuum, the screw micrometer would also be available. If there is no damping we may write

$$\gamma Mm/R^2 - ax = 2m\alpha$$

where α is the acceleration, and a the torsion coefficient, referred to x the displacement of m, at the time t, supposing the needle starts from rest, and the gravitational force is applied at $t = 0$. If at the outset we may put $X = \alpha t^2/2$

$$\gamma Mm/R^2 = X(at^2 + 4m)/t^2$$

an equation whose interesting feature is this, that if t is kept very small (which should be possible with an ocular micrometer and the achromatic fringes, a fine quartz fiber presupposed) the term involving t may be neglected and the experiment interpreted as a case of uniformly varied motion in which

$$\gamma = 2R^2\alpha/M$$

Thus for instance if $R = 5$ cm., $M = 10^3$ grams and $\gamma = 6.7 \times 10^{-8}$ and if $t = 100$ seconds is admissible, $\alpha = 1.3 \times 10^{-7}$ cm. /sec.^{2} and the distance traversed in 100 seconds would be 0.0065 cm., well measurable on the interferometer, quite so if the work is done reciprocally and the interference fringes are used individually. The theoretical error will be a minimum if m is as large as the fiber can safely carry and t as small as possible. On the other hand X is independent of m and if t is to be kept small, the result may be compensated in a large M/R^2. The measurement is thus to consist in keeping the fringes at zero by moving the micrometer screw for the small interval t during which the weight M acts. The constant γ would then follow from the micrometer reading, M and R only, all other quantities entering secondarily, as corrections. The experiment seems well worth while.

THE GENERAL CHARACTER OF SPECIFIC HEATS AT HIGH TEMPERATURES

By Walter P. White

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The wide variations presented by atomic heats at low temperatures have been brought under a single generalization, usually expressed in terms of the quantum hypothesis, in the light of which all follow a common law. This law still demands that at sufficiently high temperatures all atomic heats at constant volume should have the same constant value, approximately 5.96. The applicability of this theory, however, has been doubted; the supposition has been made that the atomic heats continue to increase with temperature, and this has been proved to be true of some metals. The present paper gives data for some non-metallic substances which show the probable existence of such an effect in every case. The substances in question are silica and various silicates. Their specific heats were determined by dropping relatively large charges (50 to 80 grams) from electric furnaces into calorimeters. Temperatures up to 1400° were used in three cases, and up to 1100° at least in most of the others.

The accuracy of the results determines the value of the evidence, and therefore needs attention.[1] (1) As to the 'accidental,' 'visible,' or continually varying errors, the discrepancy between duplicate measurements averaged under 0.5 per mille. Moreover, this small amount of discrepancy was largely accounted for by the irregularity of heat loss in dropping the hissing hot body into water, while this loss, except for its irregularities, was completely canceled out by the procedure used. (2) The unvarying, or systematic, errors were checked by using two each of practically every piece of apparatus, such as furnaces and calorimeters. Comparisons nearly always showed discrepancies under 1 per mille.

The temperature scale was established by means of one of the original Day and Sosman thermoelements, confirmed by numerous concordant metal melting points.

(3) The slowly or occasionally varying errors, the most insidious and detrimental, were checked by repeating some determinations at intervals of several months, with uniformly satisfactory results, and also by general comparisons as in table 1. Here about half the results are compared by giving their differences from a standard fictitious silicate which is approximately the mean of the whole. Of forty-five results above 0° to 100° only three, sporadic ones, those italicized, show discrepancies from a smooth curve exceeding, and then only slightly, the allowed accidental discrepancy of 1 per mille, and, what is perhaps more important, the different comparable series of values, pseudo-wollas-

tonite-diopside-enstatite, and anorthite-andesine-albite, show almost perfect concordance. The temperature distribution in the furnace, the most dangerous source of error, was studied with especial care, and the maximum irregularity detected corresponded to 0.3 per mille. The precautions against systematic error thus appear to be unusually numerous, and the conclusion seems warranted that most of the original results, for mean atomic heats over intervals, are correct to better than 3 per mille, of which less than 1 per mille is accidental.

The method of obtaining from these interval atomic heats the true atomic heats at single temperatures was demonstrably such that original *accidental*

TABLE 1

INTERVAL ATOMIC HEATS OF SILICATES, COMPARED BY BEING GIVEN AS DIFFERENCES FROM A FICTITIOUS STANDARD SILICATE

	SILICATE, MINUS STANDARD					
	0°–100°	0°–300°	0°–500°	0°–700°	0°–900°	0°–1100°
$MgSiO_3$ Amphibole	0.023	0.036	0.039	0.039	0.038	0.043
$MgSiO_3$ Pyroxene	0.036	0.059	0.084			
Diopside	0.108	0.109	0.108	0.109	0.109	0.107
$\frac{PW + Py}{2}$	0.129	0.115	0.111			
P. Woll.	0.223	0.170	0.137	0.113	0.093	0.082
Woll.				0.149	0.139	0.136
Anorthite	0.012	(008)	*0.013*	0.001	0.006	0.019
Andesine	−0.055		−0.056	−0.057	−0.053	
$\frac{Anor + Alb}{2}$	−0.048		−0.048	−0.056	−0.052	
Albite	−0.107	−0.109	−0.108	−0.113	−0.109	−0.107
Microcline	−0.096	−0.114	−0.112	−0.112	−0.116	*−0.126*
Micro. glass	0.006	*0.003*	0.013	0.017	0.021	0.063
Standard alone	4.067	4.558	4.913	5.143	5.316	5.453

errors of 1 per mille might possibly produce single errors of 1% and would be certain to produce single errors of about 5 per mille in a few cases, but would not make any *series* of results wrong by much more than the original error.

Results.—Results for seven materials are given in table 2 as true mean atomic heats. Of the substances included, silica glass is known to have an almost zero expansion coefficient, hence its atomic heat, though obtained at constant pressure, is the atomic heat at constant volume, or A_v. Cristobalite, a crystalline form of silica, shows a still lower atomic heat, and also shows it up to a considerably higher temperature. Quartz, which also is lower, has a negative coefficient of expansion from 600° to 1000° and the very low value of A_p for cristobalite may be partly or wholly a result of negative expansion. If the small thermal effect of the contraction of quartz is computed and taken

into account the result brings A_v up to its value for silica glass, within the uncertainty of the various data.

In order to interpret these results it is necessary to make allowance for the fact that according to the Kinetic Theory, as usually modified to fit the behavior of specific heats at low temperatures, A_v for these oxides is appreciably below 5.96 even at 1300°. Any high-temperature variation must appear as a variation from the value calculated from the low-temperature results. Nernst,[2] using the Nernst-Lindemann formula, has derived a special formula for the atomic heat of silica glass based on determinations from −247° to +283°, and this formula, slightly modified to agree with the present results at 283°, at which temperature Nernst had used an approximated value of Magnus', has been used to get the values given in line 5 of table 2.

These values, for 1100° and 1300°, are below the values for cristobalite by more than the probable error, which indicates that unless cristobalite changes

TABLE 2

ATOMIC HEATS. A_p

	300°	500°	700°	900°	1000°	1100°	1300°
Silica glass	4.95	5.35	5.58	5.75			
Quartz			5.46	5.66			
Cristobalite			5.55	5.67	5.72	5.78	5.86
Silica glass, formula, A_v	4.95	5.35	5.55	5.67	5.71	5.75	5.80
Albite	5.10	5.46	5.71	5.91	5.97		
Microline	5.09	5.47	5.72	5.86	5.92		
Anorthite	5.22	5.58	5.82	6.04	6.14	6.31	6.82
Platinum, A_v		6.45		6.79			6.84

very considerably its rate and probable direction of volume change near 1000°, A_v for it is increasing with temperature above the accepted theoretical value. A_v for silica glass is already well above the theoretical curve at 900°, and is probably still further above it at higher temperatures. Unfortunately, this glass could not be continuously heated above 900° without danger of crystallizing. Unlike nearly all the other series the original values for cristobalite were obviously not on a smooth curve, and were smoothed before being used to get the true heats. But none of the changes was above 1 per mille, the permitted accidental error. A very different plausible alteration gave identical results, and none which seemed reasonable gave a value at 1300° lower than 5.83 obtained from the unsmoothed data, and even this value is above the computed curve. For this curve the Debye formula, and even the original Einstein formula, give above 700° results practically identical with those in the table, so the conclusions are not dependent on the particular formula used. Since the theory applies to solid bodies only, and silica glass, being amorphous, has some of the properties of a liquid, the results for cristobalite are more sig-

nificant. The values for platinum, given for comparison, were obtained along with the silicates.

Albite and microcline have a smaller proportion of oxygen atoms than silica, and this should bring the theoretical value A_v nearer 5.96 for them than for silica. Moreover, the values of A_p given contain the thermal effect of expansion. Allowances made for these two facts bring these substances into a class with the forms of silica, and thus tend to give assurance that the silica results are not exceptional, although the data are too uncertain to add anything to the exactness of our knowledge of the variations from the theoretical curve.

Anorthite is given as a type of substances showing above 1000° a much larger increase. This increase, however, is very likely in part the effect of the latent heat from a slight amount of melting, due to a very small amount of impurity. Such effects have regularly been found in other kinds of work on silicates. But while such melting can affect the atomic heat as determined for anorthite and most other substances, which crystallize readily on cooling, it can scarcely do the same for the other silicon compounds given here, all of which are very sluggish in their changes of state, so that any slight portion of them melted at the higher temperatures would in all probability merely cool to a glass during the heat determination in the calorimeter, giving out no latent heat. Indeed, since the effect of impurities in causing melting increases rapidly with the temperature, these substances are very exceptional in their value for demonstrations at high temperatures such as that which is the subject of this paper.

Summary.—The specific heats of three forms of silica and two silicates (alkaline feldspars) determined for temperatures up to 1300°, indicate that the atomic heats at constant volume for these substances increase above theoretical value, asymptotic to 5.96, as the heats of metals have been known to do, and hence that such increase is a very general phenomenon, as has been suspected.

[1] A fuller account will be published elsewhere.

[2] Nernst, W., *Ann. Physik, Leipzig*, **36**, 1911, (430).

ON CERTAIN PROJECTIVE GENERALIZATIONS OF METRIC THEOREMS, AND THE CURVES OF DARBOUX AND SEGRE

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About a year ago appeared in these Proceedings an abstract[1] of some investigations which I was then engaged in preparing for publication. As the work progressed it expanded, and I wish to devote this note to a description

of some of my new results. The details of this discussion will appear with some of those of the former note, in extended form, in a single memoir on the general theory of surfaces.

Let P be a regular point of a non-developable surface S, and l a line in the tangent plane to S at P, but not passing through P. In my preceding note, I defined a line l' which is in the relation R to the line l with respect to the asymptotic net on S. I shall henceforth say that the lines l and l' are reciprocal to each other; they are, in fact, reciprocal polars of the osculating quadric[2] of the surface at P. The line l' passes through the point P, and does not lie in the tangent plane to S at P. If at each point of S we construct a line l' protruding from the surface, we obtain a congruence Γ'; I shall say that the congruence Γ' and the congruence Γ, formed of the lines l which are reciprocal to the lines l', are reciprocal to each other.

A projective generalization of the definition of geodesics on a surface S may easily be formulated, if the congruence of normals of the surface be replaced by a congruence Γ' composed of lines l' protruding from the surface and defined projectively in terms of the surface. There exists, in fact, a two-parameter family of curves on the surface, whose osculating planes contain the corresponding lines of the congruence Γ'. I shall, with Miss Pauline Sperry,[3] call these *union curves* of the congruence Γ'. In my previous note, I suggested that in a projective generalization of metric theorems it is desirable to preserve the important property possessed by the congruence of normals: that its developables cut the surface in a conjugate net. I also characterized a congruence Γ' having this property,[4] and uniquely determined by the surface. We may call it the *pseudo-normal congruence* of the surface, and the lines of which it is composed the *pseudo-normals* of the surface. I have recently succeeded in showing that this congruence is the most suitable substitute for the congruence of normals, in attempting to generalize the surfaces of Voss. A surface of Voss is characterized by the property, that among its geodesics exist two one-parameter families which form a conjugate net; in addition to this defining property, however, is the important one, that this conjugate net of geodesics has equal tangential invariants. I have proved that this property is merely a consequence of the fact that the developables of the congruence of normals cut the surface in a conjugate net; that is, *if there exist a conjugate net formed of union curves of a congruence* Γ', *and if this conjugate net have equal tangential invariants, then the developables of* Γ' *must cut the surface in a conjugate net.* This theorem turns out to be a consequence of the general theory of reciprocal congruences.

The definition of union curves of a congruence Γ' may be dualized so as to yield a new two-parameter family of curves, related to a congruence Γ, which we shall call *adjoint union curves*. These curves are defined as follows.[5] Let C be any curve on the surface; then the tangents to the surface which are conjugate to the tangents of C form a developable, and the edge of regression of this developable we may call the first Laplace transform of C. In fact, each

point of this edge of regression I shall call the first Laplace transform of the corresponding point of C. Let Γ be a congruence composed of lines l, one in each tangent plane of the surface. Then the curve C is said to be an *adjoint union curve* of the congruence Γ if the first Laplace transform of every point P of C lies on the corresponding line l of Γ.

A study of the relations between the union curves of a congruence Γ' and the adjoint union curves of the reciprocal congruence Γ leads to many interesting results in the general theory of surfaces and rectilinear congruences. In particular, light is thrown on a number of theorems which first appeared in my second memoir on conjugate nets.[5] In the terminology of that paper, combined with the concepts of the present note, *if Γ' be the axis congruence of a conjugate net, then the reciprocal congruence Γ is the ray congruence of the associate conjugate net.* That is, the first-mentioned conjugate net is formed of union curves of the congruence Γ', and its associate conjugate net consists of adjoint union curves of the congruence Γ which is reciprocal to Γ'. This theorem is fundamental in the study of surfaces of Voss and their projective generalization.

In general, the union curves of a congruence Γ' do not coincide with the adjoint union curves of the reciprocal congruence Γ; in fact, this can happen only on a quadric, and always does on such a surface. But on any non-ruled surface S, there are particular curves, which are union curves of certain congruences Γ', and at the same time adjoint union curves of the reciprocal congruences Γ. These curves are very important in several connections, and the property just described serves as a simple geometric characterization of them. They were first defined, in a different way, by Segre,[6] and I shall therefore call them the *curves of Segre;* they are, however, merely curves which are conjugate to certain similar curves which were previously introduced by Darboux[7] in a memoir on the contact of curves and surfaces.

The curves of Darboux were called by him *lines of quadric osculation.* He defined them as follows. Each member of the three-parameter family of quadrics which have contact of the second order with the non-ruled surface S at a regular point P cuts the surface in a curve which has a triple point at P. The three tangents at such a triple point may coincide, but, if they do, it must be in one of three directions, which Darboux calls *tangents of quadric osculation.* Associated with each of these three tangents of quadric osculation is a one-parameter family of quadrics having second-order contact with the surface at P, and cutting the surface in a curve having a triple point with coincident tangents in that particular direction of quadric osculation. There are, therefore, three one-parameter families of curves, the *curves of Darboux,* whose tangents are tangents of quadric osculation. The curve of Segre may likewise be assembled into three one-parameter families, each family being conjugate to a family of curves of Darboux.

Both Segre's and Darboux's treatments depend essentially on the notion of order of contact. It has always seemed to me to be desirable, whenever pos-

sible, to supply equivalent treatments, in such cases, which are independent of this notion; not necessarily to replace the older theorems, but to afford new theorems which are frequently of more value in practice. Moreover, it is generally difficult to define order of contact in a purely geometric way, especially projectively. Consequently the following theorem is not without interest: *on any non-ruled surface S there exist three one-parameter families of curves, the members of each family of which are union curves of a congruence* Γ' *and at the same time adjoint union curves of the reciprocal congruence* Γ. These are the curves of Segre,[6] and the curves of Darboux may be characterized as composing the three families conjugate to them.

The generalization of surfaces of Voss described above is only one of the many instances in which an important metric property of a configuration is really a particular case of a far more general projective one. I have found a number of others, connected not only with the subject of geodesics but also with apparently unrelated concepts; there appears throughout a unifying feature, however, in the notion of reciprocal congruences.

[1] Green, G. M., these PROCEEDINGS, **3**, 1917, (587–592).

[2] Wilczynski, E. J., *Trans. Amer. Math. Soc., New York*, **9**, 1908, (79–120). Cf. top of p. 83.

[3] Sperry, P., *Amer. J. Math., Baltimore*, **40**, 1918, (213–224).

[4] Green, G. M., *loc. cit.*, pp. 590–591. The congruence referred to is that generated by the lines $y\bar{y}$.

[5] Green, G. M., *Amer. J. Math., Baltimore*, **38**, 1916, (287–324).

[6] Segre, C., *Ac. dei Lincei, Rend., Rome*, (Ser. 5), 17_2, 1908, (409–411).

[7] Darboux, G., *Bull. Sci. Math., Paris*, (Ser. 2), **4**, 1880, p. 356.

THE RECTANGULAR INTERFEROMETER WITH ACHROMATIC DISPLACEMENT FRINGES IN CONNECTION WITH THE HORIZONTAL PENDULUM

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1. Introduction.—In the *Reports of the Carnegie Institute of Washington*, 1915, No. 229, Chap. I, part 2, pp. 30 et seq., I adduced a method for the application of the displacement interferometer to the horizontal pendulum with a graphic exhibit of the results obtained during a series of months. The concave mirror design by which the spectrum interference ellipses were made available showed a very satisfactory performance. The attainable accuracy was such that for moderate constants in the installation of the pendulum, an inclination of 3×10^{-4} seconds of arc should have been registered per vanishing interference fringe (ellipse), or about 10^{-3} seconds per 10^{-4} cm. of displace-

ment of the micrometer. The inclination of the line of suspending pivots was here about 1° to the vertical. A smaller angle would have correspondingly increased the sensitiveness.

The apparatus, however, required a space about two meters long between the extreme mirrors, for its installation. This is a disadvantage, since small changes of temperature in the brackets and supports as well as in the pier would interfere with the full realization of the precision of the method. The rectangular interferometer with an auxiliary mirror is thus to be preferred; for here all the necessary parts may easily be placed within a distance of one foot from the wall of the pier carrying the horizontal pendulum. If the achromatic fringes are used, these are straight and intense, so that photographic methods are available.

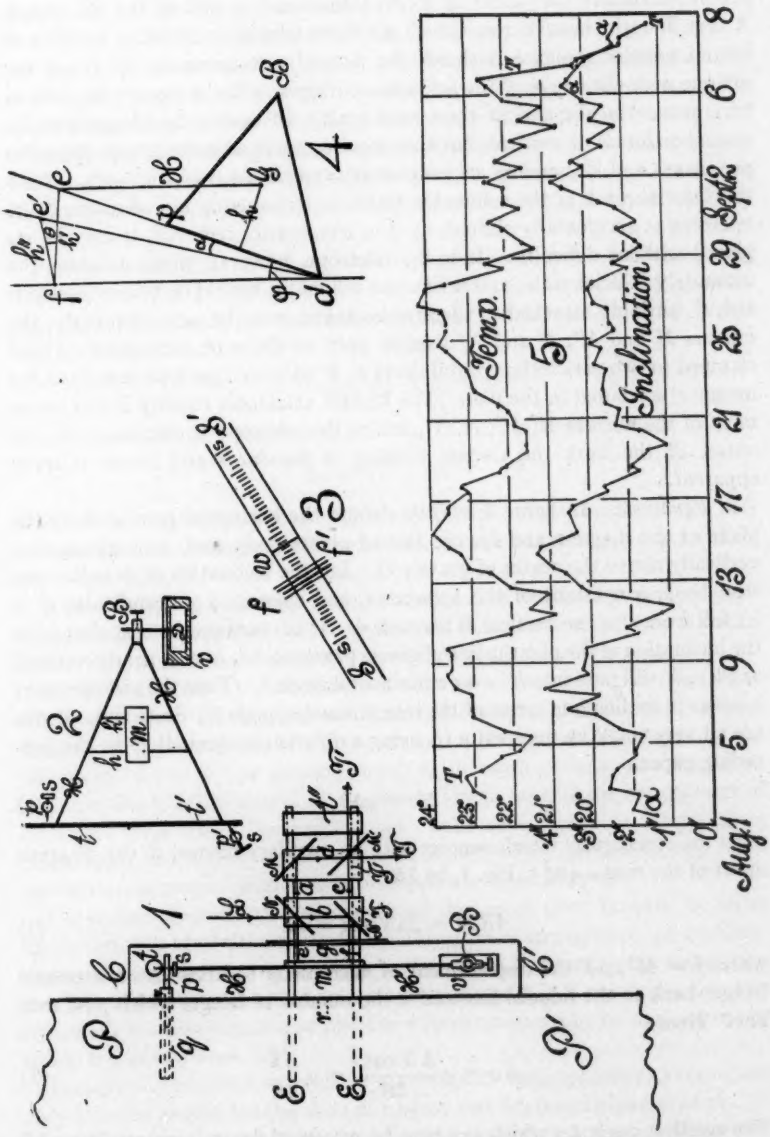
2. Apparatus.—The old horizontal pendulum made of thin steel tubing formerly described (l. c.) was again used.

Figure 1 gives a sectional plan of the pendulum installation, and figure 2 a front view of the pendulum *HH* alone, in a somewhat reduced scale. Its general shape is that of an isosceles triangle and the distance from the line of pivots *tt'* to apex about 110 cm., while the distance between pivots was 97 cm. These pivots *S*, *S'* are fine screws ending in hard steel points, which enter a glasshard steel socket (below) and a steel groove (above). The line *tt'* of the horizontal pendulum can thus be given any inclination to the vertical, while the rods *p*, *p'* which receive the screws *S*, *S'* may be moved normally to the wall of the pier *PP'*, inward or outward, and clamped to secure parallelism between the pier *PP'* and pendulum *HH'*. The apex *B* of the pendulum is also provided with a clamp, holding vane *D* submerged in an oil vat, *v*, for damping.

The whole pendulum is enclosed by a flat case, *CC'*, of tin plate provided with a plate glass window at *g*, through which the auxiliary mirror *m* of the interferometer may be seen. This is attached to one or two vertical tubes *hh'* of the pendulum, adjustably, so that it can be moved up or down, and rotated slightly about a vertical and a horizontal axis.

The interferometer consists essentially of the 4 plate glass mirrors, *M*, *M'*, *N*, *N'*, all but *M* being half silvers, the collimator (beyond *L*) and the telescope at *T* or *T'*, *t''* being a telescope support. The collimated white beam *L* is thus separated into the component rays *L*, *e*, *m*, *e*, *a*, *d*, *T*, and *L*, *b*, *f*, *m*, *f*, *c*, *T*, to be observed at either *T* or *T'*. *M'* is on a micrometer slide (not shown) with the screw normal to the face of the mirror. All mirrors should be capable of slight rotation about horizontal and vertical axes and the silvered faces all lie towards *m* for compensation of glass paths. The rays leaving *M'* for *T* must not only be accurately parallel but locally (visible as spots of light) nearly coincident; otherwise the fringes will be weak or invisible.

The telescope *T* is provided with an ocular micrometer (centimeter divided in tenth millimeters), standardized by aid of the sliding micrometer at *M'*. Moreover the image of the wide slit of the collimator adapted to the use of


Temp.
Inclination
Aug 1
5
9
13
17
21
25
29 Sept 2
8

the achromatic fringes, should be placed at right angles to them, with the ocular micrometer so placed as to read from end to end of the slit image. A very fine wire beam across the slit gives the fiducial line relative to the ocular micrometer. Figure 3 shows the general arrangement, SS' being the oblique wide slit image, ff the achromatic fringes, w the image of the fiducial wire across the slit, and ss' the ocular scale. Of course the fringes may be made horizontal or vertical; but this requires much adjustment (or else compensators) and is therefore an unnecessary complication of the work. With this fiducial mark at the collimator which is permanently out of reach, if the telescope is accidentally shifted, or it is temporarily removed, it may be replaced without difficulty. It is the telescope, however, which contains the ultimately fiducial scale, and it like the collimator should be held on a standard, t'', suitably attached to the interferometer and the pier. Similarly, the mirrors M and M', N and N' fixed in pairs to slides or carriages F, F' are clamped to two parallel horizontal tubes E, E' (3/8 inch gas pipe smoothed, for instance) anchored in the pier. The highest attainable rigidity in the placement of the mirrors M, M', N, N', and of the telescope is essential. At the outset of the work the viscous yielding of standards and braces is quite apparent.

3. Equations.—In figure 4 let pBd denote the horizontal pendulum in the plane of the diagram and dpe the line of pivots prolonged, terminating in c vertically above the center of gravity G. Let the inclination of de to the vertical be φ, a constant of the apparatus, and suppose a perpendicular h' is let fall from e to the vertical df through d. If in consequence of a change in the inclination of the pier the line of pivots passes to de', over a nearly vertical angle α, h' will pass into h'' over a horizontal angle θ. Thus the measurement consists in finding α in terms of the interferometer angle θ. Since these angles are all very small we may write (Δ being a differential symbol) as in the preceding paper.

$$\Delta\alpha = \varphi\Delta\theta \tag{1}$$

In the rectangular interferometer with an auxiliary mirror, if the distance apart of the rays a and c, Fig. 1, be $2R$

$$4R\Delta\theta = 2\Delta N\cos i = n\lambda \tag{2}$$

where $i = 45°$, ΔN the displacement of micrometer to bring the achromatic fringes back to the fiducial line and n the number of fringes which pass that line. Hence

$$\Delta\alpha = \varphi\frac{\Delta N\cos i}{2R} = \varphi\frac{n\lambda}{4R} \tag{3}$$

The smallest angle $\Delta\alpha$ which can thus be measured depends essentially on $2R$ the breadth of the ray parallelogram. There would be no difficulty in making this as long as the line from tt' to B of the horizontal pendulum, i.e. over a meter;

but this would necessitate two mirrors m one at each end. For the present purposes, I preferred to use apparatus which I had at hand in which $2R$ was but 10 cm. and a single mirror could be used at m. Nevertheless if $n = 1$, the limit of angle measurable, if $\varphi = 0.0175$ or one degree is $\Delta\alpha = 5 \times 10^{-8}$ radian per fringe; i.e. about 0.01 second of arc. With an ocular micrometer and well produced achromatic fringes there is no difficulty in estimating 1/10 fringe, so that the limiting angle here is to be a few thousandths of a second, even if $\varphi = 1°$, which may also be reduced. By making $2R = 100$ cm. one should therefore be able to reach 0.0001 second per tenth fringe breadth, if φ is 1°.

Finally on using the ocular micrometer for moderately sized fringes of say one scale part (0.1 millimeter fringes in the ocular) the case is equally promising. A comparison of the two displacements ΔN at M' and Δe of the fringes in the ocular, showed $\Delta e/\Delta N = 265$. Hence

$$\Delta\alpha = \varphi\Delta e \cos i/265 \times 2R$$

If $\Delta e/\Delta N = 10^{-2}$ (one scale part) and $2R = 10$ cm. etc., as above,

$$\Delta\alpha = 2.7 \times 10^{-8}$$

or 0.005″ per scale part of the ocular micrometer. A few tenths of this may be estimated on the scale. The sensitiveness will be ten times greater if $2R$ is a meter.

4. Observations.—The interferometer was installed with rather smaller fringes than instanced above and therefore with less sensitiveness, as the inclination of the pier in a heated laboratory would probably run into seconds of arc in the lapse of time. For this reason R was also satisfactory at its small value of $2R = 10$ cm. The angle φ was directly measureable, as the inclination of the line joining the points of the pivots to the plumb line. Under these circumstances $\Delta\alpha = 0.9\ \Delta e$ seconds, roughly, the tenth millimeters of the ocular scale are about 1/100 second of arc in relation to $\Delta\alpha$ and the fringes were of about the same size. There would have been no difficulty in making them much larger and therefore more sensitive as they were clear and strong. The end of the compound pendulum was damped in lubricating oil.

The earlier observations were discarded; but even after January 14 (after which time the apparatus worked smoothly), there are instances of displacement within the apparatus requiring readjustment. These betray themselves in a lack of coincidence of the two wide slit images, Fig. 3, or of the cross wire w (the slit is really superfluous and the collimator lens may be so placed as to widen the illuminated field).

Though observations were made continuously since January, I can here only give the graphs for the days of August and September, just passed. In figure 5, the lower curve shows the apparent change of inclination of the pier in seconds of arc; the upper curve the corresponding temperature (in degrees centigrade) of the basement laboratory room. Observations were usually made at 10 a.m. and 6 p.m. Between August 1 and 21, the resemblance of

the two curves is marked. After that the gross resemblance is no longer so striking, but it nevertheless remains in the details and there is a mere lack of equivalence, quantitatively. The question therefore arises where this temperature discrepancy has its seat. It can hardly be in the interferometer, where the parts are of the same metal except in the (virtually) tetrahedral bracket, consisting of the rods, E, E', and the brace downward from t'' to r in the pier. For here one part $t''r$ is of iron and the other from EE' downward to r, of brick. There might thus be differential expansion; but the interferometer would not be sensitive to this motion, of which I convinced myself by bearing down hard at t'' with the hands. No adequate displacement of fringes resulted. Hence it appears probable that what is observed is the warping of the pier, etc., as a result of the inward progress of the successive isotherms through it, beginning at the parts least protected against changes of temperature by the surrounding house walls. At all events this temperature feature is so serious that a few tenths of a degree centigrade can not be overlooked.